Research Article

Existence of Mild and Classical Solutions for Nonlocal Impulsive Integrodifferential Equations in Banach Spaces with Measure of Noncompactness

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We study the existence of mild and classical solutions are proved for a class of impulsive integrodifferential equations with nonlocal conditions in Banach spaces. The main results are obtained by using measure of noncompactness and semigroup theory. An example is presented.

1. Introduction

In this present paper, we are concerned with the existence of mild and classical solutions are proved for a class of impulsive integrodifferential equations with nonlocal conditions:

$$
\begin{aligned}
u'(t) &= Au(t) \\
&+ f\left(t, u(t), \int_0^t k(t,s,u(s))\,ds, \int_0^a h(t,s,u(s))\,ds\right), \\
&\hspace{4cm} t \in [0,a], \quad t \neq t_i \\
u(0) &= u_0 + g(u), \\
\Delta u(t_i) &= I_i(u(t_i)), \quad i = 1, 2, \ldots, p,
\end{aligned}
$$

(1)

where A is the infinitesimal generator of a C_0-semigroup $T(t)$ in a Banach space X and $f : [0,a] \times X \times X \to X$, $k : [0,a] \times [0,a] \times X \to X$, $h : [0,a] \times [0,a] \times X \to X$, $0 < t_1 < t_2 < t_3 < \cdots < t_p < a$ and $I_i : X \to X$, $i = 1, 2, \ldots, p$ are impulsive functions and $g : PC([0,a]; X) \to X$, $u_0 \in X$,

and X is a real Banach space with norm $\|\cdot\|$. $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$, $u(t_i^+), u(t_i^-)$ denote the right and left limits of u at t_i, respectively.

Integrodifferential equations are important for investigating some problems raised from natural phenomena. They have been studied in many different aspects. The theory of semigroups of bounded linear operators is closely related to the solution of differential and integrodifferential equations in Banach spaces. In recent years, this theory has been applied to a large class of nonlinear differential equations in Banach spaces. We refer to the papers [1–5] and the references cited therein. Based on the method of semigroups, existence, and uniqueness of mild, strong, and classical solutions of semilinear evolution equations were discussed by Pazy [6]. In [7], Xue studied the semilinear nonlocal differential equations with measure of noncompactness in Banach spaces. Lizama and Pozo [8] investigated the existence of mild solutions for semilinear integrodifferential equation with nonlocal initial conditions by using Hausdorff measure of noncompactness via a fixed point.

In recent years, the impulsive differential equations have been an object of intensive investigation because of the wide

possibilities for their applications in various fields of science and technology as theoretical physics, population dynamics, economics, and so forth; see [9–13]. The study of semilinear nonlocal initial problem was initiated by Byszwski [14, 15] and the importance of the problem lies in the fact that it is more general and yields better effect than the classical initial conditions. Therefore it has been extensively studied under various conditions on the operator A and the nonlinearity f by several authors [13, 16–18].

Byszwski and Lakshmikantham [19] prove the existence and uniqueness of mild solutions and classical solutions when f and g satisfy Lipschitz-type conditions. Ntouyas and Tsamotas [20, 21] study the case of compactness conditions of f and $T(t)$. Zhu et al. [22] studied the existence of mild solutions for abstract semilinear evolution equations in Banach spaces. In [23], Liu discussed the existence and uniqueness of mild and classical solutions for the impulsive semilinear differential evolution equation. In [24], the authors studied the existence of mild solutions to an impulsive differential equation with nonlocal conditions by applying Darbo-Sadovskii's fixed point theorem. In recent paper [25], Ahmad et al. studied nonlocal problems of impulsive integrodifferential equations with measure of noncompactness. For some more recent results and details, see [26–29].

Motivated by the above-mentioned works, we derive some sufficient conditions for the solutions of integrodifferential equations (1) combining impulsive conditions and nonlocal conditions. Our results are achieved by applying the Hausdorff measure of noncompactness and fixed point theorem. In this paper, we denote by $N = \sup\{\|T(t)\| : t \in [0, a]\}$. Without loss of generality, we let $u_0 = 0$.

2. Preliminaries

Let $(X, \|\cdot\|)$ be a real Banach space. We denote by $C([0, a]; X)$ the space of X-valued continuous functions on $[0, a]$ with the norm $\|x\| = \sup \|x(t)\|, t \in [0, a]$ and by $L^1(0, a; X)$ the space of X-valued Bochner integrable functions on $[0, a]$ with the norm $\|f\|_{L^1} = \int_0^a \|f(t)\| dt$.

We put $J_i = (t_i, t_{i+1}), i = 1, 2, \ldots, p$. In order to define the mild solution of problem (1), we introduce the following set.

$PC([0, a]; X) = \{u : [0, a] \to X : u$ is continuous on $J_i, i = 0, 1, 2, \ldots, p$ and the right limit $u(t_i)$ exists, $i = 1, 2, \ldots, p\}$.

Definition 1. A function $u \in PC([0, a]; X)$ is a mild solution of (1) if

$$
\begin{aligned}
u(t) = & T(t) [u_0 - g(u)] \\
& + \int_0^t T(t - s) f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau, \right. \\
& \qquad\qquad\qquad \left. \int_0^a h(s, \tau, u(\tau)) d\tau\right) ds \\
& + \sum_{0 < t_i < t} T(t - t_i) I_i(u(t_i)), \quad 0 \le t \le a.
\end{aligned}
\tag{2}
$$

The Hausdorff measure of noncompactness $\beta_{\mathbb{Y}}(B)$ is defined by $\beta_{\mathbb{Y}}(B) = \inf\{r > 0, B$ can be covered by finite number of balls with radii $r\}$ for bounded set B in a Banach space Y.

Lemma 2 (see [30]). *Let $\mathbb{Y}$ be a real Banach space and $B; C \subseteq \mathbb{Y}$ be bounded, with the following properties:*

(1) *B is precompact if and only if $\beta_{\mathbb{X}}(B) = 0$;*

(2) *$\beta_{\mathbb{Y}}(B) = \beta_{\mathbb{Y}}(\overline{B}) = \beta_{\mathbb{Y}}(convB)$, where $\overline{B}$ and $convB$ mean the closure and convex hull of B, respectively;*

(3) *$\beta_{\mathbb{Y}}(B) \le \beta_{\mathbb{Y}}(C)$, where $B \subseteq C$;*

(4) *$\beta_{\mathbb{Y}}(B + C) \le \beta_{\mathbb{Y}}(B) + \beta_{\mathbb{Y}}(C)$, where $B + C = \{x + y : x \in B, y \in C\}$;*

(5) *$\beta_{\mathbb{Y}}(B \cup C) \le \max\{\beta_{\mathbb{Y}}(B), \beta_{\mathbb{Y}}(C)\}$;*

(6) *$\beta_{\mathbb{Y}}(\lambda B) \le |\lambda| \beta_{\mathbb{Y}}(B)$ for any $\lambda \in \mathbb{R}$;*

(7) *if the map $Q : D(Q) \subseteq \mathbb{Y} \to \mathbb{Z}$ is Lipschitz continuous with constant k, then $\beta_{\mathbb{Z}}(QB) \le k\beta_{\mathbb{Y}}(B)$ for any bounded subset $B \subseteq D(Q)$, where $\mathbb{Z}$ be a Banach space;*

(8) *$\beta_{\mathbb{Y}}(B) = \inf\{d_{\mathbb{Y}}(B, C); C \subseteq \mathbb{Y}$ is precompact$\} = \inf\{d_{\mathbb{Y}}(B, C); C \subseteq \mathbb{Y}$ is finite valued$\}$, where $d_{\mathbb{Y}}(B, C)$ means the nonsymmetric (or symmetric) Hausdorff distance between B and C in $\mathbb{Y}$;*

(9) *if $\{W_n\}_{n=1}^{+\infty}$ is decreasing sequence of bounded closed nonempty subsets of $\mathbb{Y}$ and $\lim_{n \to \infty} \beta_{\mathbb{Y}}(W_n) = 0$, then $\bigcap_{n=1}^{+\infty} W_n$ is nonempty and compact in $\mathbb{Y}$.*

The map $Q : W \subseteq \mathbb{Y} \to \mathbb{Y}$ is said to be a $\beta_{\mathbb{Y}}$-contraction if there exists a positive constant $k < 1$ such that $\beta_{\mathbb{Y}}(Q(B)) \le k\beta_{\mathbb{Y}}(B)$ for any bounded closed subset $B \subseteq W$, where $\mathbb{Y}$ is a Banach space.

Lemma 3 (Darbo-Sadovskii [30]). *If $W \subseteq Y$ is bounded closed and convex, the continuous map $Q : W \to W$ is a $\beta_{\mathbb{Y}}$-contraction, then the map Q has at least one fixed point in W.*

Lemma 4 (see [2]). *If $W \subseteq PC([0, a]; X)$ is bounded, then $\alpha(W(t)) \le \alpha_{PC}(W)$ for all $t \in [0, a]$, where $W(t) = \{u(t) : u \in W\} \subseteq X$. Furthermore if W is equicontinuous on each interval J_i of $[0, a]$, then $\alpha(W(t))$ is continuous on $[0, a]$, and $\alpha_{PC}(W) = \sup\{\alpha(W(t)) : t \in [0, a]\}$.*

Lemma 5 (see [3]). *If $\{u_n\}_{n=1}^\infty \subset L^1(0, a; X)$ is uniformly integrable, then $\alpha(\{u_n(t)\}_{n=1}^\infty)$ is measurable and*

$$
\alpha\left(\left\{\int_0^t u_n(s) ds\right\}_{n=1}^\infty\right) \le 2 \int_0^t \alpha\left(\{u_n(s)\}_{n=1}^\infty\right) ds. \tag{3}
$$

Lemma 6 (see [31]). *If the semigroup $T(t)$ is equicontinuous and $\eta \in L^1(0, a; R^+)$, then the set*

$$
\left\{ t \longrightarrow \int_0^t T(t - s) u(s) ds; u \in L^1(0, K; R^+), \right.
$$

$$
\left. \|u(s)\| \le \eta(s), \text{ for a.e } s \in [0, a] \right\}
\tag{4}
$$

is equicontinuous on [0,a].

Lemma 7 (see [23]). *If W is bounded, then for each $\varepsilon > 0$, there is a sequence $\{u_n\}_{n=1}^{\infty} \subseteq W$, such that $\alpha(W) \leq 2\alpha(\{u_n\}_{n=1}^{\infty}) + \varepsilon$.*

3. g Is Compact

In this section, we give the existence results of nonlocal integrodifferential equation (39). Here we list the following hypotheses.

(Hg_1) The c_0 semigroup $T(t)$, $0 \leq t \leq a$, generated by A is equicontinuous.

(Hg_2) (i) $g : PC([0, a]; X) \rightarrow X$ is continuous and compact.

(ii) There exists $M > 0$ such that $\|g(u)\| \leq M$, for all $u \in PC([0, a]; X)$, and $\widehat{k}(s) = \max\{1, \int_0^s k(s, \tau)d\tau\}$.

(I) Let $I_i : X \rightarrow X$ be continuous, compact map and there are nondecreasing functions $l_i : R^+ \rightarrow R^+$, satisfying $\|I_i(x)\| \leq l_i(\|x\|)$, $i = 1, 2, \ldots, p$.

(Hf_1) There exists a continuous function $a_k : [0, a] \times [0, a] \rightarrow [0, \infty)$ and a nondecreasing continuous function $\Omega_k : R^+ \rightarrow R^+$ such that $\|k(t, s, x)\| \leq a_k(t, s)\Omega_k(\|x\|)$ for all $x \in X$ a.e. $t, s \in [0, a]$. And there exists at least one mild solution to the following scalar equation:

$$
\begin{aligned}
m(t) = {} & MN \\
& + M \int_0^t a_f(s) \Omega_f\big(m(s), a_k(t, s)\Omega_k m(s), \\
& \qquad\qquad b_h(t, s)\Omega_h m(s)\big) ds \\
& + M \sum_{i=1}^p l_i(m(t)), \quad t \in [0, a].
\end{aligned}
\tag{5}
$$

(Hf_2) (i) $f(\cdot, \cdot, x, y)$ is measurable for $x, y \in X$, $f(t, \cdot, \cdot, \cdot)$ is continuous for a.e. $t \in [0, a]$.

(ii) There exist a function $a_f(\cdot) \in L^1(0, a, R^+)$ and an increasing continuous function $\Omega_f : R^+ \rightarrow R^+$ such that $\|f(t, x, y, z)\| \leq a_f(t)\Omega_f(\|x\|, \|y\|, \|z\|)$ for all $x, y, z \in X$ and a.e. $t \in [0, a]$.

(iii) $f : [0, a] \times X \times X \rightarrow X$ is compact.

(Hf_3) There exists a function $\eta \in L^1(0, a; R^+)$ such that for any bounded $D \subset X$,

$$
\beta(f(t, D_1, D_2, D_3)) \leq \eta(t)(\beta(D_1) + \beta(D_2) + \beta(D_3))
\tag{6}
$$

for a.e. $t \in [0, a]$ and for any bounded subset $D \subset PC([0, a], X)$.

Here we let $\widehat{k}(s) = \int_0^s k(s, \tau)d\tau$ and $(1 + 2\widehat{k_1}(s) + 2\widehat{k_2}(s)) \leq Q$.

Theorem 8. *Assume that the hypotheses (Hg_1), (Hg_2), I, (Hf_1), and (Hf_2) are satisfied; then the nonlocal impulsive problem (1) has at least one mild solution.*

Proof. Let $m(t)$ be a solution of the scalar equation (5); the map $K : PC([0, a]; X) \rightarrow PC([0, a]; X)$ is defined by

$$
(Ku)(t) = (K_1 u)(t) + (K_2 u)(t)
\tag{7}
$$

with

$$
\begin{aligned}
(K_1 u)(t) = {} & T(t) g(u) \\
& + \int_0^t T(t - s) f\bigg(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau, \\
& \qquad\qquad \int_0^a h(s, \tau, u(\tau)) d\tau\bigg) \\
(K_2 u)(t) = {} & \sum_{0 < t_i < t} T(t - t_i) I_i(u(t_i)),
\end{aligned}
\tag{8}
$$

for all $t \in [0, a]$.

It is easy to see that the fixed point of K is the mild solution of nonlocal impulsive problem (1).

From our hypotheses, the continuity of K is proved as follows.

For this purpose, we assume that $u_n \rightarrow u$ in $PC([0, a]; X)$. It comes from the continuity of k and h that $k(s, \tau, u_n(\tau)) \rightarrow k(s, \tau, u(\tau))$ and $h(s, \tau, u_n(\tau)) \rightarrow h(s, \tau, u(\tau))$, respectively.

By Lebesgue convergence theorem,

$$
\begin{aligned}
\int_0^s k(s, \tau, u_n(\tau)) d\tau & \longrightarrow \int_0^s k(s, \tau, u(\tau)) d\tau, \\
\int_0^a h(s, \tau, u_n(\tau)) d\tau & \longrightarrow \int_0^a h(s, \tau, u(\tau)) d\tau.
\end{aligned}
\tag{9}
$$

Similarly we have

$$
\begin{aligned}
& f\bigg(s, u_n(s), \int_0^s k(s, \tau, u_n(\tau)) d\tau, \int_0^a h(s, \tau, u_n(\tau)) d\tau\bigg) \\
& \longrightarrow f\bigg(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau, \int_0^a h(s, \tau, u(\tau)) d\tau\bigg),
\end{aligned}
\tag{10}
$$

for all $s \in [0, a]$. Consider

$$
\begin{aligned}
& \|Ku_n - Ku\| \\
& \leq N \|g(u_n) - g(u)\| \\
& \quad + N \int_0^t \bigg(\bigg\| f\bigg(s, u_n(s), \int_0^s k(s, \tau, u_n(\tau)) d\tau,
\end{aligned}
$$

$$\int_0^a h(s, \tau, u_n(\tau)) \, d\tau \Bigg)$$

$$- f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.$$

$$\left. \int_0^a h(s, \tau, (\tau)) \, d\tau \right) \Bigg\| \Bigg) ds \tag{11}$$

$$+ \sum_{i=1}^p M \left\| I_i(u_n(t_i)) - I_i(u(t_i)) \right\| \longrightarrow 0,$$

as $n \to \infty$. So $Ku_n \to Ku$ in PC$([0,a]; X)$. That is, K is continuous.

We denote $W_0 = \{u \in \text{PC}([0,a]); X), \|u(t)\| \le m(t) \text{ for all } t \in [0,a]\}$; then $W \subseteq \text{PC}([o,a]; X)$ is bounded and convex.

Define $W_1 = \overline{\text{conv}} K(W_0)$, where $\overline{\text{conv}}$ means that the closure of the convex hull in PC$([0,a]; X)$.

For any $u \in K(W_0)$, we know that

$$\|(u)(t)\| \le MN$$

$$+ N \int_0^t a_f(s) \, \Omega_f \left(m(s), a_k(t,s) \, \Omega_k m(s), \right.$$

$$\left. b_h(t,s) \, \Omega_h m(s) \right) ds \tag{12}$$

$$+ M \sum_{i=1}^p l_i(m(t)), \quad t \in [0,a],$$

and by (Hf_2), $W_1 \subset W_0$.

From the Arzela-Ascoli theorem, to prove the compactness of K, we can prove that $K_1 u : u \in W_0$ is equicontinuous and $K_1 u(t) \subset X$ is precompact for $t \in [0,a]$:

$$\|K_1 u(t+\sigma) - K_1 u(t)\|$$

$$= \Bigg\| T(t+\sigma) g(u)$$

$$+ \int_0^{t+\sigma} T(t+\sigma-s) f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.$$

$$\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) ds$$

$$- \left[T(t) g(u) + \int_0^t T(t-s) f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) ds \right] \Bigg\|$$

$$\le \left\| [T(t+\sigma) - T(u)] g(u) \right\|$$

$$+ \int_t^{t+\sigma} \|T(t+\sigma-s)\| \left\| f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) \right\| ds$$

$$+ \int_0^t \|T(t+\sigma-s) - T(t-s)\|$$

$$\times \left\| f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) \right\| ds$$

$$\le MN \|T(\sigma) - I\|$$

$$+ N \int_0^{t+\sigma} a_f(s) \, \Omega_f \left(\|u(s)\|, \int_0^s a_k(s, \tau) \, \Omega_k(\|u(s)\|) \, d\tau, \right.$$

$$\left. \int_0^a b_h(s, \tau) \, \Omega_h(\|u(s)\|) \, d\tau \right) ds$$

$$+ N \int_0^t \left\| [T(\sigma) - I] f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) \right\| ds. \tag{13}$$

Since f is compact, $MN\|T(\sigma) - I\|$ and $\|[T(\sigma) - I] f(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau, \int_0^a h(s, \tau, u(\tau)) d\tau)\| \to 0$ as $\sigma \to 0$ uniformly for $s \in [0,a]$ and $u \in \text{PC}([0,a]; X)$. This implies that for any $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$, there exist a $\delta > 0$ such that

$$\|[T(\sigma) - I]\| \le \varepsilon_1,$$

$$\Bigg\| [T(\sigma) - I]$$

$$\times f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) \Bigg\|$$

$$\le \varepsilon_2, \tag{14}$$

for $0 \le \sigma < \delta$ and all $u \in \text{PC}([0,a]; X)$. Therefore

$$\|[T(\sigma) - I]\|$$

$$+ \Bigg\| [T(\sigma) - I]$$

$$\times f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) \Bigg\|$$

$$\le \varepsilon_1 + \varepsilon_2 = \varepsilon. \tag{15}$$

We know that

$$\|K_1 u(t+\sigma) - K_1 u(t)\|$$

$$\le MN\varepsilon + N\Omega_f \left(m(s), \|a_k(a,a)\|, \Omega_k(m(s)), \right. \tag{16}$$

$$\left. \|b_h(a,a)\|, \Omega_h(m(s)) \right) + N\varepsilon,$$

for $0 \le \sigma < \delta$ and all $u \in \text{PC}([0,a]; X)$. So $\{K_1 u : u \in W_0\}$ is equicontinuous.

The set $\{T(t - s)f(s, u(s)), \int_0^s k(s, \tau, u(\tau))d\tau, \int_0^a h(s, \tau, u(\tau))d\tau); t, s \in [0, a], u \in \text{PC}([0, a]; X)\}$ is precompact as f is compact and $T(\cdot)$ is a C_0-semigroup.

So $K_1 u(t) \subset X$ is precompact as $K_1 u(t) \subset t \ \overline{\text{conv}}\{T(t - s)f(s, u(s)), \int_0^s k(s, \tau, u(\tau))d\tau, \int_0^a h(s, \tau, u(\tau))d\tau); s \in [0, t], u \in \text{PC}([0, a]; X)\}$ for all $t \in [0, a]$. W_1 is equicontinuous on each interval J_i of $[0, a]$. For $t_i \le t < t+\sigma \le t_{i+1}, i = 1, 2, \ldots, p$, we have, using the semigroup properties

$$\|(K_2 u)(t + \sigma) - (K_2 u)(t)\|$$

$$\le \left\| \sum_{0 < t_i < t+\sigma} T(t + \sigma - t_i) I_i(x(t_i)) \right.$$

$$\left. - \sum_{0 < t_i < t} T(t + \sigma - t_i) I_i(x(t_i)) \right\|$$

$$+ \left\| \sum_{0 < t_i < t} T(t + \sigma - t_i) I_i(x(t_i)) \right.$$

$$\left. - \sum_{0 < t_i < t} T(t - t_i) I_i(x(t_i)) \right\|$$

$$\le \sum_{0 < t_i < t+\sigma} \|T(t + \sigma - t_i) I_i(x(t_i))\|$$

$$+ \sum_{0 < t_i < t} \|T(t + \sigma - t_i) I_i(x(t_i)) - T(t - t_i) I_i(x(t_i))\|,$$

$$(17)$$

which follows that $\{K_2 u : u \in W_0\}$ is equicontinuous on each J_i due to the equicontinuous of $T(t)$ and hypotheses (I). Therefore, $W_1 \subset \text{PC}([0, a]; X)$ is bounded closed convex nonempty and equicontinuous on each interval J_i, $i = 0, 1, 2, \ldots, p$.

We define $W_{n+1} = \overline{\text{conv}}K(W_n)$, for $n = 1, 2, \ldots, p$. From above we know that $\{W_n\}_{n=1}^{\infty}$ is a decreasing sequence of bounded, closed, convex nonempty subsets in $\text{PC}([0, a]; X)$ and equicontinuous on each J_i, $i = 1, 2, \ldots, p$.

Now for $n \ge 1$ and $t \in [0, a]$, $W_n(t)$ and $K(W_n(t))$ are bounded subsets of X. Hence for any $\varepsilon > 0$, there is a sequence $\{u_k\}_{k=1}^{\infty} \subset W_n$ such that (see, e.g., [2, page 125])

$$\beta(W_{n+1}(t))$$

$$= \beta(KW_n(t))$$

$$\le 2\beta\left(T(t) g\left(\{u_k\}_{k=1}^{\infty}\right)\right)$$

$$+ 2\beta\left(\int_0^t T(t - s) f\left(s, \{u_k(s)\}_{k=1}^{\infty}, \int_0^s k\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau,\right.\right.$$

$$\left.\left. \int_0^a h\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right) ds\right)$$

$$+ 2\beta\left(\sum_{i=1}^{p} T(t - t_i) I_i\{u_k(t_i)\}_{k=1}^{\infty}\right) + \varepsilon,$$

$$(18)$$

for $t \in [0, a]$.

From the compactness of g and I_i, by Lemmas 2 and 5 and (Hf_3), we have

$$\beta(W_{n+1}(t))$$

$$\le 2\beta\left(\int_0^t T(t - s) f\left(s, \{u_k(s)\}_{k=1}^{\infty}, \int_0^s k\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau,\right.\right.$$

$$\left.\left. \int_0^a h\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right) ds\right) + \varepsilon$$

$$\le 4N \int_0^t \beta\left(f\left(s, \{u_k(s)\}_{k=1}^{\infty}, \int_0^s k\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau,\right.\right.$$

$$\left.\left. \int_0^a h\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right)\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) + \beta\left(\int_0^s k\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right)\right.$$

$$\left. + \beta\left(\int_0^a h\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right)\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)\right.$$

$$+ 2\int_0^s \beta\left(k\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right)$$

$$\left. + 2\int_0^a \beta\left(h\left(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}\right) d\tau\right)\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)\right.$$

$$+ 2\left(\int_0^s \eta_1(s, \tau) d\tau\right)\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)$$

$$\left. + 2\left(\int_0^a \eta_2(s, \tau) d\tau\right)\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) + 2\widehat{k_1}(s)\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)\right.$$

$$\left. + 2\widehat{k_2}(s)\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right)\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\left(\beta(W_n(s)) + 2\widehat{k_1}(s)\beta(W_n(s))\right.$$

$$\left. + 2\widehat{k_2}(s)\beta(W_n(s))\right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s)\beta(W_n(s))\left(1 + 2\widehat{k_1}(s) + 2\widehat{k_2}(s)\right) ds + \varepsilon$$

$$\le 4NQ \int_0^t \eta(s)\beta(W_n(s)) ds + \varepsilon.$$

$$(19)$$

Since $\varepsilon > 0$ is arbitrary, it follows from the above inequality that

$$\beta(W_{n+1}(t)) \le 4NQ \int_0^t \eta(s) \beta_{\text{PC}}(W_n(s)) ds, \qquad (20)$$

for all $t \in [0, a]$. Since W_n is decreasing for n, we define $f_n(t) = \lim_{n \to \infty} \beta(W_n(t))$ for all $t \in [0, a]$. From (20), we have

$$f_{n+1}(t) \le 4NQ \int_0^t \eta(s) f_n(s) \, ds, \tag{21}$$

for $t \in [0, a]$, which implies that $f_n(t) = 0$ for all $t \in [0, a]$. By Lemma 4, we know that

$$\lim_{n \to \infty} \beta_{PC}(W_n) = 0. \tag{22}$$

Using Lemma 2, we also know that

$$W = \bigcap_{n=1}^{\infty} W_n \tag{23}$$

is convex, compact, and nonempty in $PC([0, a]; X)$ and $K(W) \subset W$.

By the famous Schauder's fixed point theorem, there exists at least one mild solution u of the problem (1), where $u \in W$ is a fixed point of the continuous map K. □

4. g Is Lipschitz

In this section, we discuss the problem (1) when g is Lipschitz continuous and I_i, $i = 1, 2, \ldots, p$ is not compact. We replace hypotheses (Hf_2), (I) by

$(Hf_2)'$ There is a constant $L \in (0, 1/M)$ such that $\|g(u) - g(v)\| \le L\|u - v\|_{PC}$ for all $u, v \in PC([0, a]; X)$.

(I') There exists $L_i > 0$, $i = 1, 2, \ldots, p$, such that $\|I_i(u) - I_i(v)\| \le L_i\|u - v\|$, for all $u, v \in X$.

Theorem 9. *Assume that the hypotheses* (Hg_1), $(Hf_2)'$, (I'), (Hf_1)–(Hf_3) *are satisfied. Then the nonlocal impulsive problem* (1) *has at least one mild solution on* $[0, a]$, *provided that*

$$ML + 4NQ \int_0^t \eta(s) \, ds + 2N \sum_{i=1}^p L_i < 1. \tag{24}$$

Proof. Define the operator K : $PC([0, a]; X) \to PC([0, a]; X)$ by

$$(Ku)(t) = (K_1u)(t) + (K_2u)(t) + (K_3u)(t). \tag{25}$$

With

$$(K_1u)(t) = T(t) g(u),$$

$$(K_2u)(t) = \int_0^t T(t - s) f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) \, d\tau, \right.$$

$$\left. \int_0^a h(s, \tau, u(\tau)) \, d\tau \right) ds,$$

$$(K_3u)(t) = \sum_{0 < t_i < t} T(t - t_i) I_i(u(t_i)),$$

$$\tag{26}$$

for all $u \in PC([0, a]; X)$.

Define $W_0 = \{u \in PC([0, a]; X) : \|u(t)\| \le m(t) \text{ for all } t \in [0, a]\}$, and let $W = \overline{KW_0}$.

Then from the proof of Theorem 8, we know that W is a bounded closed convex and equicontinuous subset of $PC([0, a]; X)$ and $KW \subset W$. We will prove that K is β_{PC}-contraction on W. Then Darbo-Sadovskii fixed point theorem can be used to get a fixed point of K in W, which is a mild solution of (1).

We first show that K_1 is Lipschitz on $PC([0, a]; X)$.

In fact, take $u, v \in PC([0, a]; X)$ arbitrary. Then by $(Hf_2)'$, we have

$$\left\|(K_1u)(t) - (K_2u)(t)\right\|$$
$$\le M \|g(u) - g(v)\| \le ML\|u - v\|_{PC} \quad \forall t \in [0, a]. \tag{27}$$

It follows that $\|K_1u - K_1v\| \le ML\|u - v\|_{PC}$ for all $u, v \in PC([0, a]; X)$. That is, K_1 is Lipschitz with Lipschitz constant ML.

Next, for every bounded subset $B \subset W$, for any $\varepsilon > 0$, there is a sequence $\{u_k\}_{k=1}^{\infty} \subset B$ such that $\beta(K_2B(t)) \le 2\beta(\{K_2u_k(t)\}_{k=1}^{\infty}) + \varepsilon$, for $t \in [0, a]$. Since B and K_2B are equicontinuous, we get from Lemmas 2 and 5 and (Hf_3) that

$$\beta(K_2B(t))$$

$$\le 2\beta \left(\int_0^t T(t - s) f\left(s, \{u_k(s)\}_{k=1}^{\infty}, \int_0^s k(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) ds \right) + \varepsilon$$

$$\le 4N \int_0^t \beta \left(f\left(s, \{u_k(s)\}_{k=1}^{\infty}, \int_0^s k(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau, \right.\right.$$

$$\left.\left. \int_0^a h(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) \right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s) \left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) + \beta\left(\int_0^s k(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) \right.$$

$$\left. + \beta\left(\int_0^a h(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) \right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s) \left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) + 2\int_0^s \beta\left(k(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) \right.$$

$$\left. + 2\int_0^a \beta\left(h(s, \tau, \{u_k(\tau)\}_{k=1}^{\infty}) \, d\tau \right) \right) ds + \varepsilon$$

$$\le 4N \int_0^t \eta(s) \left(\beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) \right.$$

$$\left. + 2\left(\int_0^s \eta_1(s, \tau) \, d\tau \right) \beta\left(\{u_k(s)\}_{k=1}^{\infty}\right) \right.$$

$$+ 2 \left(\int_0^a \eta_2 (s, \tau) \, d\tau \right) \beta \left(\{ u_k (s) \}_{k=1}^\infty \right) \right) ds + \varepsilon$$

$$\leq 4N \int_0^t \eta (s) \left(\beta \left(\{ u_k (s) \}_{k=1}^\infty \right) + 2 \widehat{k_1} (s) \beta \left(\{ u_k (s) \}_{k=1}^\infty \right) \right.$$

$$\left. + 2 \widehat{k_2} (s) \beta \left(\{ u_k (s) \}_{k=1}^\infty \right) \right) ds + \varepsilon$$

$$\leq 4N \int_0^a \eta (s) \left(\beta_{\mathrm{PC}} \left(\{ u_k \}_{k=1}^\infty \right) + 2 \widehat{k_1} (s) \beta_{\mathrm{PC}} \left(\{ u_k \}_{k=1}^\infty \right) \right.$$

$$\left. + 2 \widehat{k_2} (s) \beta_{\mathrm{PC}} \left(\{ u_k \}_{k=1}^\infty \right) \right) ds + \varepsilon$$

$$\leq 4N \int_0^a \eta (s) \beta_{\mathrm{PC}} (B) \left(1 + 2 \widehat{k_1} (s) + 2 \widehat{k_2} (s) \right) ds + \varepsilon$$

$$\leq 4NQ \int_0^a \eta (s) \, ds \beta_{\mathrm{PC}} (B) + \varepsilon, \tag{28}$$

for $t \in [0, a]$. Since $\varepsilon > 0$ arbitrary, we have

$$\beta_{\mathrm{PC}} (K_2 B) \leq 4NQ \int_0^a \eta (s) \beta_{\mathrm{PC}} (B) \, ds, \tag{29}$$

for any bounded subset $B \subset W$.

$$\beta (K_3 B(t)) \leq 2N \sum_{i=1}^p \beta \left(I_i \{ u_k (t_i) \}_{k=1}^\infty \right)$$

$$\leq 2N \sum_{i=1}^p L_i \beta \left(\{ u_k (t_i) \}_{k=1}^\infty \right) \tag{30}$$

$$\beta_{\mathrm{PC}} (K_3 B) \leq 2N \sum_{i=1}^p L_i \beta_{\mathrm{PC}} (B),$$

for any subset $B \subset W$; due to Lemma 2, (29), and (30), we have

$$\beta_{\mathrm{PC}} (KB)$$

$$= \beta_{\mathrm{PC}} (K_1 B) + \beta_{\mathrm{PC}} (K_2 B) + \beta_{\mathrm{PC}} (K_3 B)$$

$$\leq ML \beta_{\mathrm{PC}} (B) + 4NQ \int_0^a \eta (s) \beta_{\mathrm{PC}} (B) \, ds + 2N \sum_{i=1}^p L_i \beta_{\mathrm{PC}} (B)$$

$$\leq \left(ML + 4NQ \int_0^a \eta (s) \, ds + 2N \sum_{i=1}^p L_i \right) \beta_{\mathrm{PC}} (B). \tag{31}$$

From (24), we know that K is β_{PC}-contraction on W. By Lemma 3, there is a fixed point u of K in W, which is a mild solution of problem (1). □

5. Classical Solutions

To study the classical solutions, let us recall the following result.

Lemma 10. *Assume that $u_0 \in D(A), q_i \in D(A), i = 1, 2, \dots, p$, and that $f \in C^1([0, a] \times X, X)$.*
Then the impulsive differential equation

$$u' (t) = Au (t) + f (t, u (t)), \quad t \in [0, a], \ t \neq t_i$$

$$u (0) = u_0, \tag{32}$$

$$\Delta u (t) = q_i, \quad i = 1, 2, \dots, p,$$

has a unique classical solution $u(\cdot)$ which satisfies, for $t \in [0, a]$,

$$u (t)$$

$$= T (t) u (0) = \int_0^t T (t - s) f (s, u (s)) \, ds + \sum_{0 < t_i < t} T (t - t_i) q_i. \tag{33}$$

Now we make the following assumption.
(H^1) There exists a function $\eta \in L^1(0, a; R^+)$ such that for any bounded $D \subset X$,

$$\beta \left(f (t, D_1, D_2, D_3) \right) \leq \eta (t) \left(\beta (D_1) + \beta (D_2) + \beta (D_3) \right), \tag{34}$$

or a.e. $t \in [0, a]$ and for any bounded subset $D \subset PC([0, a], X)$.

Theorem 11. *Let (H^1) be satisfied and $u(\cdot)$ a mild solution of the problem (1). Assume that $u(0) \in D(A), I_i(u(t_i)) \in D(A), i = 1, 2, \dots, p$ and that $f \in C^1([0, a] \times X, X)$. Then $u(0)$ gives rise to a classical solution of the problem (1).*
If $u(\cdot)$ is a uniquely determined mild solution, then it gives rise to a unique classical solution.

Proof. We can define $q_i = I_i(u(t_i)), i = 1, 2, \dots, p$.
From Lemma 10,

$$v' (t) = Av (t) + f \left(t, v (t), \int_0^t k (t, s, v (s)) \, ds, \right.$$

$$\left. \int_0^a h (t, s, v (s)) \, ds \right) \quad t \in [0, a], \ t \neq t_i$$

$$v (0) = u_0 + g (u),$$

$$\Delta v (t) = q_i, \quad i = 1, 2, \dots, p \tag{35}$$

has a unique classical solution $v(\cdot)$ which satisfies, for $t \in [0, a]$,

$$v (t) = T (t) \left[u_0 - g (u) \right]$$

$$+ \int_0^t T (t - s) f \left(s, v (s), \int_0^s k (s, \tau, v (\tau)) \, d\tau, \right.$$

$$\left. \int_0^a h (s, \tau, v (\tau)) \, d\tau \right) ds \tag{36}$$

$$+ \sum_{0 < t_i < t} T (t - t_i) I_i (u (t_i)), \quad 0 \leq t \leq a.$$

Now, $u(\cdot)$ is a mild solution of the problem (1), so that we get for $t \in [0, a]$,

$$u(t) = T(t)[u_0 - g(u)]$$

$$+ \int_0^t T(t-s) f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau,\right.$$

$$\left. \int_0^a h(s, \tau, u(\tau)) d\tau\right) ds \qquad (37)$$

$$+ \sum_{0 < t_i < t} T(t - t_i) I_i(u(t_i)), \quad 0 \le t \le a.$$

Thus we get

$$v(t) - u(t)$$

$$= \int_0^t T(t-s) \left(f\left(s, v(s), \int_0^s k(s, \tau, v(\tau)) d\tau,\right.\right.$$

$$\int_0^a h(s, \tau, v(\tau)) d\tau\right)$$

$$- f\left(s, u(s), \int_0^s k(s, \tau, u(\tau)) d\tau,\right.$$

$$\left.\left. \int_0^a h(s, \tau, u(\tau)) d\tau\right)\right) ds, \qquad (38)$$

which gives, by (H^1) and an application of Gronwall's inequality, $\|u - v\|_{PC} = 0$.

This implies that $u(\cdot)$ gives rise to a classical solution and completes the proof. □

6. Example

Let Ω be a bounded domain in R_n with smooth boundary $\partial\Omega$, and $X = L^2(\Omega)$. Consider the following nonlinear integrodifferential equation in X:

$$\frac{\partial u(t, y)}{\partial t} = \Delta u(t, y)$$

$$+ \frac{\gamma_1 u(t, y)}{(1+t)(1+t^2)} \sin(u(t, y))$$

$$+ \int_0^t \frac{\gamma_2 u(s, y)}{(1+t)(1+t^2)(1+s)^2} ds \qquad (39)$$

$$+ \int_0^a \frac{\gamma_3 u(s, y)}{(1+t)(1+t^2)(1+s)^2(1+s^2)^2} ds,$$

$$u(t_i^+, y) - u(t_i^-, y) = I_i(u(t_i, y)), \quad i = 1, 2, \ldots, p, \quad (40)$$

with nonlocal conditions

$$u(0) = u_0(y) + \int_\Omega \int_0^a h_1(t, y) \log\left(1 + |u(t, s)|^{1/2}\right) dt\, ds,$$

$$y \in \Omega, \qquad (41)$$

or

$$u(0) = u_0(y) + \gamma_4 u(t, y), \quad y \in \Omega, \qquad (42)$$

where $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \in R, h_1(t, y) \in PC([0, a]; \overline{\Omega})$. Set $A = \Delta, D(A) = W^{2,2}(\Omega) \bigcap W_0^{1,2}(\Omega)$,

$$f\left(t, u(t), \int_0^t k(t, s, u(s)) ds, \int_0^a h(t, s, u(s)) ds\right)(y)$$

$$= \frac{\gamma_1 u(t, y)}{(1+t)(1+t^2)} \sin(u(t, y))$$

$$+ \int_0^t \frac{\gamma_2 u(s, y)}{(1+t)(1+t^2)(1+s)^2} ds \qquad (43)$$

$$+ \int_0^a \frac{\gamma_3 u(s, y)}{(1+t)(1+t^2)(1+s)^2(1+s^2)^2} ds.$$

Define nonlocal conditions

$$g(u)(y)$$

$$= u_0(y) + \int_\Omega \int_0^a h_1(t, y) \log\left(1 + |u(t, s)|^{1/2}\right) dt\, ds \qquad (44)$$

or

$$g(u)(y) = u_0(y) + \gamma_4 u(t, y). \qquad (45)$$

It is easy to see that A generates a compact C_0-semigroup in X, and

$$\left\| f\left(t, u(t), \int_0^t k(t, s, u(s)) ds, \int_0^a h(t, s, u(s)) ds\right)\right\|$$

$$\le |\gamma| (1, \|u\|, \|j\|, \|q\|), \qquad (46)$$

$$|\gamma| = \max\{|\gamma_1|, |\gamma_2|, |\gamma_3|\},$$

where

$$j = \int_0^t \frac{\gamma_2 u(s, y)}{(1+t)(1+t^2)(1+s)^2} ds,$$

$$k(t, s, u(s)) = \frac{\gamma_2 u(s, y)}{(1+t)(1+t^2)(1+s)^2}, \qquad (47)$$

$$q = \int_0^a \frac{\gamma_3 u(s, y)}{(1+t)(1+t^2)(1+s)^2(1+s^2)^2} ds,$$

$$h(t, s, u(s)) = \frac{\gamma_3 u(s, y)}{(1+t)(1+t^2)(1+s)^2(1+s^2)^2},$$

and $\|k(t, s, u(s))\| \le |\gamma_2|(1 + \|u\|), \|h(t, s, u(s))\| \le |\gamma_3|(1 + \|u\|), \|I_i(u)\| \le l_i(\|u\|), i = 1, 2, \ldots, p$.

For nonlocal conditions (44), $\|g(u)\| \le a(\text{mes}(\Omega)) \max_{t \in [0,a], y \in \Omega} |h_1(t, y)|[\|u\| + (\text{mes}(\Omega))^{1/2}], u \in PC([0, a]; X)$, and g is compact example of [18].

For nonlocal conditions (45),

$$\|g(u_1) - g(u_2)\| \le |\gamma_4| \|u_1 - u_2\|. \qquad (48)$$

Hence, g is Lipschitz. Furthermore, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ and a can be chosen such that (24) is also satisfied. Obviously, it satisfies all the assumptions given in our Theorem 9; the problem has at least one mild solution in $\text{PC}([0, a]; L^2(\Omega))$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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